ON/OFF control trajectory computation for steady state reaching in batches Petri nets

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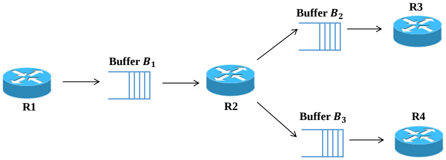
Motivation

Objective

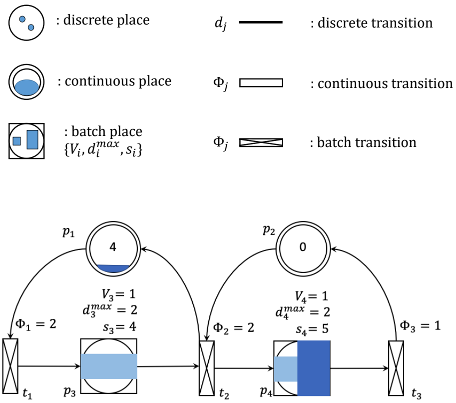
Control the transient behavior to reach a target steady state from a given initial state in hybrid /discrete event systems.



1. Variable delay for the transferred data in a buffer.
2. Accumulation in a buffer.
3. Limited capacity in a buffer, i.e., length × density.

Generalized batches Petri nets could be used as hybrid model with timed event dynamics.

Generalized batches Petri nets

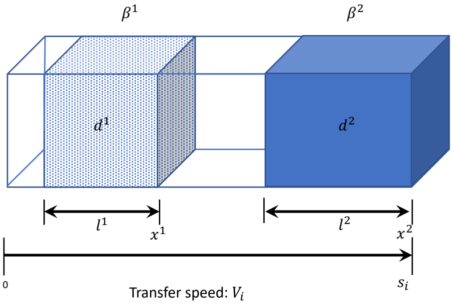


A generalized batches Petri net (GBPN) is a 6-tuple $N = (P, T, Pre, Post, \gamma, Time)$ where:

- $P = P^D \cup P^C \cup P^B$
- $T = T^D \cup T^C \cup T^B$
- Pre and $Post$: $(P^D \times T \to \mathbb{N}) \cup ((P^C \cup P^B) \times T \to \mathbb{R}_{\geq 0})$
- $\gamma : P^B \to \mathbb{R}^3_{>0}$, $\gamma(p_i) = (V_i, d_i^{max}, s_i)$
- $Time : T \to \mathbb{R}_{\geq 0}$, $Time(t_j) = d_j$ if $t_j \in T^D$; $Time(t_j) = \Phi_j$ if $t_j \in T^C \cup T^B$

Batch

A batch is a group of discrete entities, characterized by three continuous variables: a length, a density and a head position.



Definition

Inside batch place p_i, a batch $\beta_i^k(\tau) = (l_i^k(\tau), d_i^k(\tau), x_i^k(\tau))$.

1. $l_i^k(\tau) \in \mathbb{R}_{\geq 0}$ is the length.
2. $d_i^k(\tau) \in \mathbb{R}_{\geq 0}$ is the density.
3. $x_i^k(\tau) \in \mathbb{R}_{\geq 0}$ is the head position.

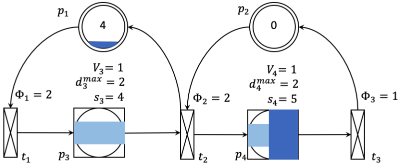
1. A batch β_i^k with $d_i^k = d_i^{\max}$, is called as dense batch.
2. A batch β_i^k with $x_i^k = s_i$, is called as output batch.

Marking and marking quantity

The marking of a GBPN at time τ, $\boldsymbol{m}(\tau) = [m_1(\tau) m_2(\tau) \ldots m_n(\tau)]^T$, is a function that assigns to each discrete, continuous and batch place a nonnegative integer, a nonnegative real number and a series of batches $m_i(\tau) = \{\beta_i^1(\tau), \ldots, \beta_i^r(\tau)\}$, respectively.

The marking quantity vector $\boldsymbol{q} = \mu(\boldsymbol{m}) \in \mathbb{R}^n_{\geq 0}$ associated with marking $\boldsymbol{m}$ is:

$$q_i = \begin{cases} m_i & \text{if } p_i \in P^D \cup P^C, \\ \sum_{\beta_i^k \in m_i} l_i^k \cdot d_i^k & \text{if } p_i \in P^B. \end{cases}$$



The marking:

$m_1 = 4$. $m_2 = 0$.
$m_3 = \{\beta_3^1\}, \beta_3^1 = (4, 1, 4)$.
$m_4 = \{\beta_4^1, \beta_4^2\}, \beta_4^1 = (2, 1, 2), \; \beta_4^2 = (3, 2, 5)$.

The marking quantity:

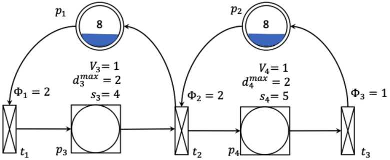
$q_1 = 4$; $q_2 = 0$.
$q_3 = l_3^1 \cdot d_3^1 = 4$.
$q_4 = l_4^1 \cdot d_4^1 + l_4^2 \cdot d_4^2 = 8$.

Firing flow vectors

The instantaneous firing flow vector at time τ is denoted as $\boldsymbol{\varphi}(\tau) \in \mathbb{R}^{|T|}$, where $\varphi_j(\tau) \leq \Phi_j$ represents the firing quantity of transition t_j by time unit.

The controlled firing flow vector, denoted as $\boldsymbol{u}(\tau)$, with $0 \leq u_j(\tau) \leq \Phi_j$, controls the evolution of the net.

Remark: in an invariant behavior state, both firing flow vectors remain constant (between timed events).



At $\tau_0 = 0$, transition t_1 is enabled while t_2 and t_3 are not enabled.

The instantaneous firing flow vector is: $\boldsymbol{\varphi}(\tau_0) = [2\ 0\ 0]^T$.

The controlled firing flow vector is: $\boldsymbol{u}(\tau_0) = [u_1(\tau_0)\ 0\ 0]^T$ with $0 \leq u_1(\tau_0) \leq 2$.

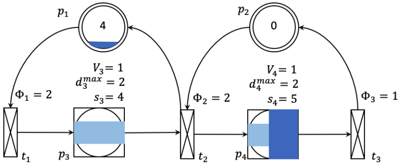
Steady state and regular marking

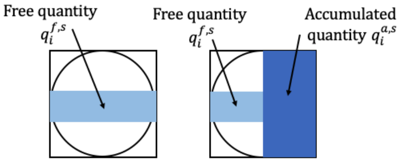
Let $\langle N, \boldsymbol{m}_0 \rangle$ be a cGBPN with $P^D = T^D = \emptyset$. The net is in a steady state at time τ_s if for $\tau \geq \tau_s$ the marking $\boldsymbol{m}^s$ and the instantaneous firing flow vector $\boldsymbol{\varphi}^s$ remain constant.

In a steady state $(\boldsymbol{m}^s, \boldsymbol{\varphi}^s)$ with $\boldsymbol{\varphi}^s > 0$, the marking of a batch place p_i takes one of the following regular form:

1. A single batch: $m_i^s = \{\beta_i^o\}$ with $\beta_i^o = (s_i, d_i^o, s_i)$.
2. Two batches: $m_i^s = \{\beta_i^e, \beta_i^o\}$ with $\beta_i^e = (l_i^e, d_i^e, l_i^e)$, $\beta_i^o = (l_i^o, d_i^{\max}, s_i)$.

In a steady state, the marking quantity of batch place p_i is called the steady marking quantity, denoted as q_i^s with $q_i^s = q_i^{f,s} + q_i^{a,s}$.





Control trajectory

Objective

Control the transient behavior to reach a target steady state from a given initial state in generalized batches Petri nets.

Control trajectory

Given a cGBPN system $\langle N, \boldsymbol{m}_0 \rangle$, a control trajectory to reach a steady state $(\boldsymbol{m}^s, \boldsymbol{\varphi}^s)$ is given as $(\boldsymbol{u}^0, \tau_0), (\boldsymbol{u}^1, \tau_1), \cdots, (\boldsymbol{u}^i, \tau_i), \cdots, (\boldsymbol{u}^s, \tau_s)$ such that the controlled firing flow vector $\boldsymbol{u}^i$ is applied from date τ_i until the next event date τ_{i+1} and, $\boldsymbol{u}^s = \boldsymbol{\varphi}^s$.

For a cGBPN without discrete nodes:

$$\boldsymbol{q}^s = \boldsymbol{q}(\tau_s) = \boldsymbol{q}^0(\tau_0) + \boldsymbol{C} \cdot \left(\int_{\tau_0}^{\tau_1} \boldsymbol{u}^0(\rho) d\rho + \cdots + \int_{\tau_i}^{\tau_{i+1}} \boldsymbol{u}^i(\rho) d\rho + \cdots + \int_{\tau_s}^{\infty} \boldsymbol{u}^s(\rho) d\rho \right),$$

where $\int_{\tau_i}^{\tau_{i+1}} \boldsymbol{u}^i(\rho) \cdot d\rho$ denotes the sum of firing quantity of continuous or batch transitions during time interval $[\tau_{i-1}, \tau_i]$ and, $C \cdot \int_{\tau_s}^{\infty} \boldsymbol{u}^s(\rho) d\rho = 0$.

Control strategy

Given a cGBPN system $\langle N, \boldsymbol{m}_0 \rangle$ and a reachable steady state $(\boldsymbol{m}^s, \varphi^s)$.

Assumptions

- A1 No discrete nodes ($P^D = T^D = \emptyset$).
- A2 The steady firing flow vector is positive ($\varphi^s > 0$).
- A3 The net is conservative.

Control strategy

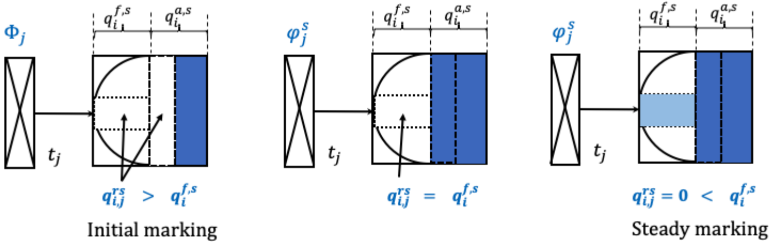
1. ON_{ss}: maximize $u_j(\tau)$ to φ_j^s if t_j is enabled with $\exists\ p_k \in t_j^\bullet : q_{k,j}^{rs}(\boldsymbol{m}) \leq q_k^{f,s}$.
2. ON_{max}: maximize $u_j(\tau)$ to Φ_j if t_j is enabled with $\nexists\ p_k \in t_j^\bullet : q_{k,j}^{rs}(\boldsymbol{m}) \leq q_k^{f,s}$.
3. OFF: $u_j(\tau) = 0$ if t_j is not enabled or if t_j is enabled with $\exists\ p_i \in {}^\bullet t_j : q_i(\tau) < q_i^s$ and $\exists\ p_k \in t_j^\bullet : q_{k,j}^{rs}(\boldsymbol{m}) \leq q_k^{f,s}$.

Control strategy

The condition for switching the controlled flow of transition t_j from the maximal flow to the steady flow is:

$$q^{rs}_{i,j}(\boldsymbol{m}) \leq q^{f,s}_i,$$

where $q^{rs}_{i,j}(\boldsymbol{m})$ is the minimum remaining quantity that enters a place p_i for reaching the steady marking quantity vector.



Algorithm for controlling the transient behavior

Algorithm Computation of control trajectory

Input: A cGBPN $\langle N, \boldsymbol{m}_0 \rangle$, a reachable steady state $(\boldsymbol{m}^s, \boldsymbol{\varphi}^s)$ and the steady firing quantity vector $\boldsymbol{z}^s$.
Output: Control trajectory $(\boldsymbol{u}^0, \tau_0), (\boldsymbol{u}^1, \tau_1), \cdots$.

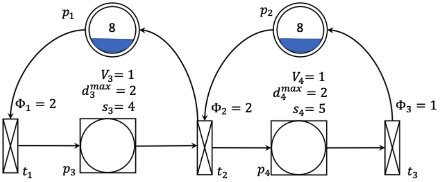
1: $\boldsymbol{q}^0 = \mu(\boldsymbol{m}_0)$, $\boldsymbol{m}^0 = \boldsymbol{m}_0$, $\boldsymbol{u}^0 = 0$, $\boldsymbol{z}^0 = 0$, $\tau_0 = 0$, $i = 0$
2: **while** $\boldsymbol{m}^i \neq \boldsymbol{m}^s$ **do**
3: Determine $T_{\mathcal{N}}(\boldsymbol{m}^i)$, $T_L(\boldsymbol{m}^i)$,$T_Z(\boldsymbol{m}^i)$, $T_{ZL}(\boldsymbol{m}^i)$, $P_{\emptyset}(\boldsymbol{m}^i)$, $P_F(\boldsymbol{m}^i)$ $S_L(\boldsymbol{m}^i)$, $S_E(\boldsymbol{m}^i)$, $S_{ZE}(\boldsymbol{m}^i)$
4: Solve the following LPP: $\max 1^T \cdot \boldsymbol{u}^i \quad s.t.$

$$\begin{cases}
\text{(a) } 0 \leq u_j^i \leq \Phi_j & \forall t_j \in T \\
\text{(b) } 0 \leq u_j^i \leq \varphi_j^s & \forall t_j \in T_Z(\boldsymbol{m}^i) \\
\text{(c) } u_j^i = 0 & \forall t_j \in T_{\mathcal{N}}(\boldsymbol{m}^i) \cup T_{ZL}(\boldsymbol{m}^i) \\
\text{(d) } C(p_k, \cdot) \cdot \boldsymbol{u}^i \leq 0 & \forall p_k \in P_F(\boldsymbol{m}^i) \\
\text{(e) } C(p_k, \cdot) \cdot \boldsymbol{u}^i \geq 0 & \forall p_k \in P_{\emptyset}(\boldsymbol{m}^i) \\
\text{(f) } C(p_k, \cdot) \cdot \boldsymbol{u}^i = 0 & \forall p_k \in S_{ZE}(\boldsymbol{m}^i) \\
\text{(g) } \mathrm{Post}(p_k, \cdot) \cdot \boldsymbol{u}^i \leq V_k \cdot d_k^{\max} & \forall p_k \in P^B \\
\text{(h) } \mathrm{Pre}(p_k, \cdot) \cdot \boldsymbol{u}^i \leq V_k \cdot d_k^{\mathrm{out}} & \forall p_k \in P^B
\end{cases}$$

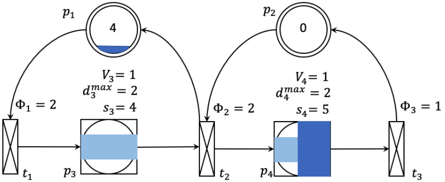
5: Determine all the next timed events from considered ones, select the nearest at time τ_{i+1}
6: Determine the new marking $\boldsymbol{m}^{i+1}$, the marking quantity vector $\boldsymbol{q}^{i+1}$ and the current firing quantity vector $\boldsymbol{z}^{i+1}$
7: $\mathrm{i} = \mathrm{i} + 1$
8: **end while**
9: $\boldsymbol{u}^i = \boldsymbol{\varphi}^s$
10: Return $(\boldsymbol{u}^0, \tau_0), (\boldsymbol{u}^1, \tau_1), \cdots$

Example

The initial state is $\boldsymbol{m}_0 = [8\ 8\ \emptyset\ \emptyset]^T$.



The steady state is $\boldsymbol{m}^s = [4\ 0\ \{(4,1,4)\}\ \{(2,1,2),(3,2,5)\}]^T$, $\boldsymbol{\varphi}^s = [1\ 1\ 1]^T$.

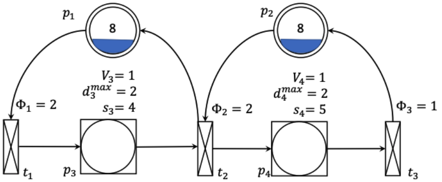


$q_3^s = q_3^{f,s} = 4.$

$q_4^s = 8$ with
$q_4^{f,s} = 2$ and $q_4^{a,s} = 6.$

Example

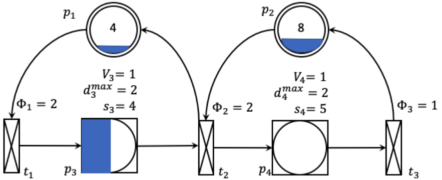
At $\tau_0 = 0$, the marking is $\boldsymbol{m}_0 = [8\ 8\ \emptyset\ \emptyset]^T$.



Conditions:
$q^{rs}_{3,1}(\tau_0) = 12 > q_3^{f,s} = 4$.

The controlled firing flow vector is $\boldsymbol{u}^0 = [2\ 0\ 0]^T$.

At $\tau_1 = 2$, the marking is $\boldsymbol{m}^1 = [4\ 8\ \{(2,2,2)\}\ \emptyset]^T$.

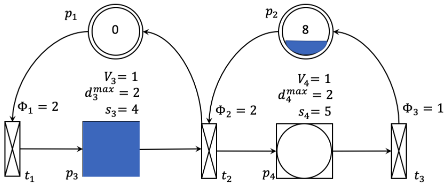


Events: $q_1(\tau_1) = q_1^s$,
$q_3(\tau_1) = q_3^s$.
Conditions:
$q^{rs}_{3,1}(\tau_1) = 8 > q_3^{f,s} = 4$.

The controlled firing flow vector is $\boldsymbol{u}^1 = [2\ 0\ 0]^T$.

Example

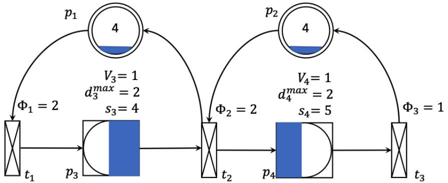
At $\tau_2 = 4$, the marking is $\boldsymbol{m}^2 = [0\ 8\ \{(4,2,4)\}\ \emptyset]^T$.



Events: $q_1(\tau_2) = 0$,
$q_3(\tau_2) = 8$.
Conditions:
$q^{rs}_{3,1}(\tau_2) = q_3^{f,s} = 4$.
$q^{rs}_{4,2}(\tau_2) = 8 > q_4^{f,s} = 2$.

The controlled firing flow vector is $\boldsymbol{u}^2 = [0\ 2\ 0]^T$.

At $\tau_3 = 6$, the marking is $\boldsymbol{m}^3 = [4\ 4\ \{(2,2,4)\}\ \{(2,2,2)\}]^T$.



Events: $q_1(\tau_3) = q_1^s$,
$q_3(\tau_3) = q_3^s$.
Conditions:
$q^{rs}_{4,2}(\tau_3) = 4 > q_4^{f,s} = 2$.

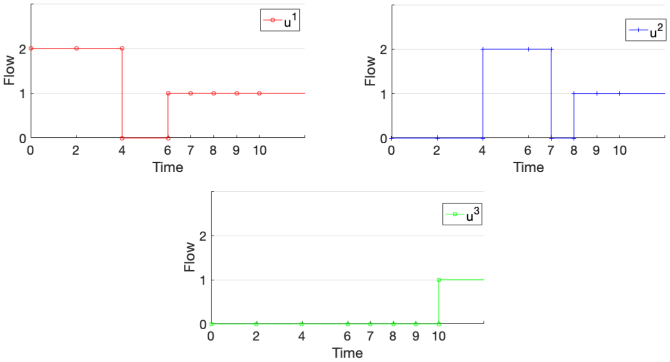
The controlled firing flow vector is $\boldsymbol{u}^3 = [1\ 2\ 0]^T$.

At $\tau_7 = 10$, the steady state is reached.

Example

Control trajectory: $(\boldsymbol{u}^0, 0), (\boldsymbol{u}^1, 2), (\boldsymbol{u}^2, 4), (\boldsymbol{u}^3, 6), (\boldsymbol{u}^4, 7), (\boldsymbol{u}^5, 8), (\boldsymbol{u}^6, 9), (\boldsymbol{u}^7, 10)$.

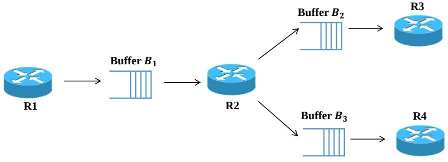
- The delay is $\tau_s = 10\ u.t.$
- The number of events is 7.

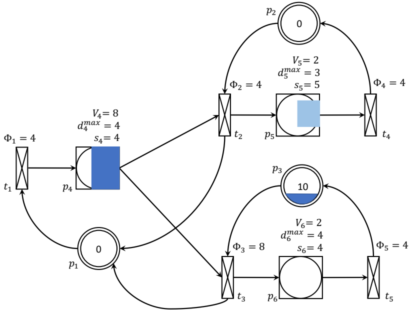


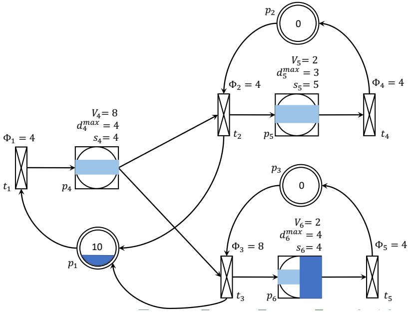
Case study

The initial state is
$\boldsymbol{m}_0 = [0\ 0\ 10\ \{(3,4,4)\}\ \{(2.5,2,5)\}\ \emptyset]^T$.

The steady state is
$\boldsymbol{m}^s = [10\ 0\ 0\ \{(4,0.5,4)\}$
$\{(5,1,5)\}\ \{(2,1,2),(2,4,4)\}]^T$,
$\boldsymbol{\varphi}^s = [4\ 2\ 2\ 2\ 2]^T$.







Case study

Using maximal flow based ON/OFF control strategy, the steady state is reached from the initial one within:

- the delay $\tau_s = 2.5\ u.t.$
- the number of events 5.

Using steady flow based ON/OFF control strategy developed in our previous work, the steady state is reached from initial one within:

- the delay $\tau_s' = 5\ u.t.$
- the number of events 5.

The transient behavior delay is reduced while the number of events is equal.

Conclusions

We propose a strategy for controlling the transient behavior of a dynamical system represented by a hybrid formalism, a generalised batches Petri net.

This control strategy, called maximal flow based ON/OFF control strategy, allows the system to reach a target steady state from an initial state.

It is based on:

1. a pure event driven dynamics,
2. control variables defined on the firing flow of transitions,
3. the maximization of firing flows to their maximal values.

Thanks for your attention!