Multi-robot path planning using Petri nets

01

Introduction

Introduction

Last-Mile Delivery


Unmanned vehicle


Delivery robot


Delivery drones

Automated Warehouse


Sorting machine


Stacker robot


Shuttle carrier system

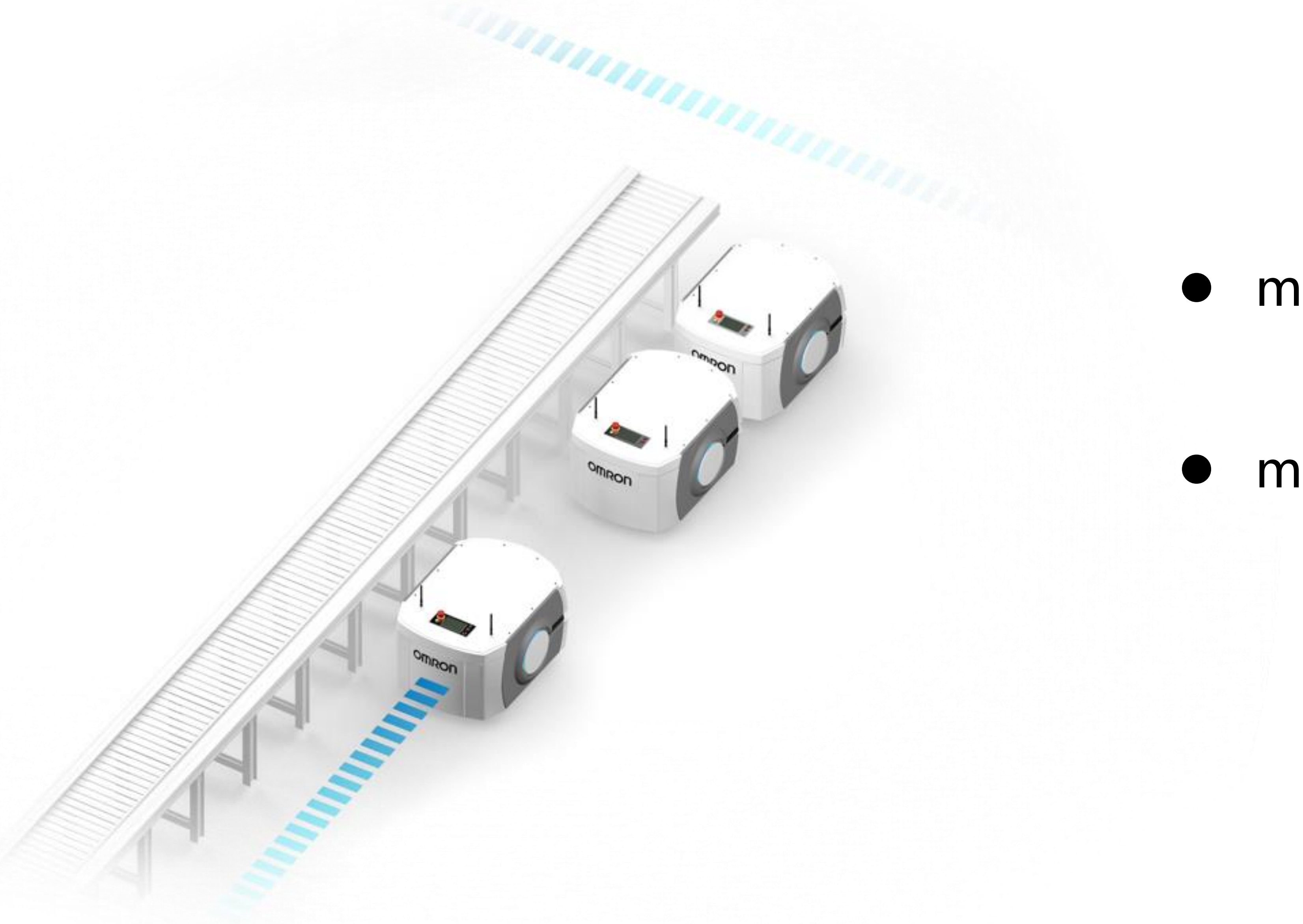

Folklift


Sorting AGV Capability


AGV

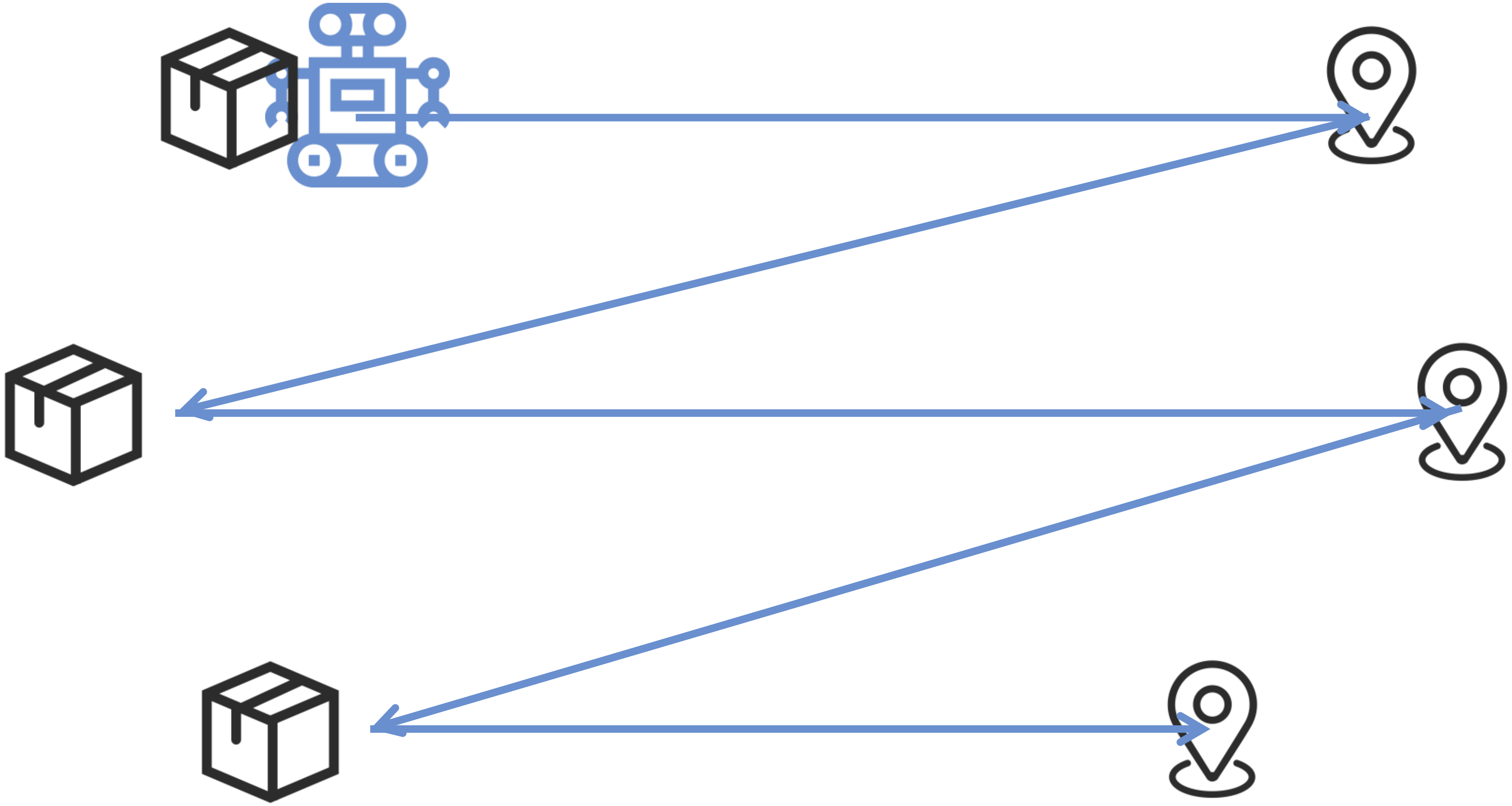

Warehouse
Surveillance Car

Introduction



- more complex tasks
- more efficiently

Introduction



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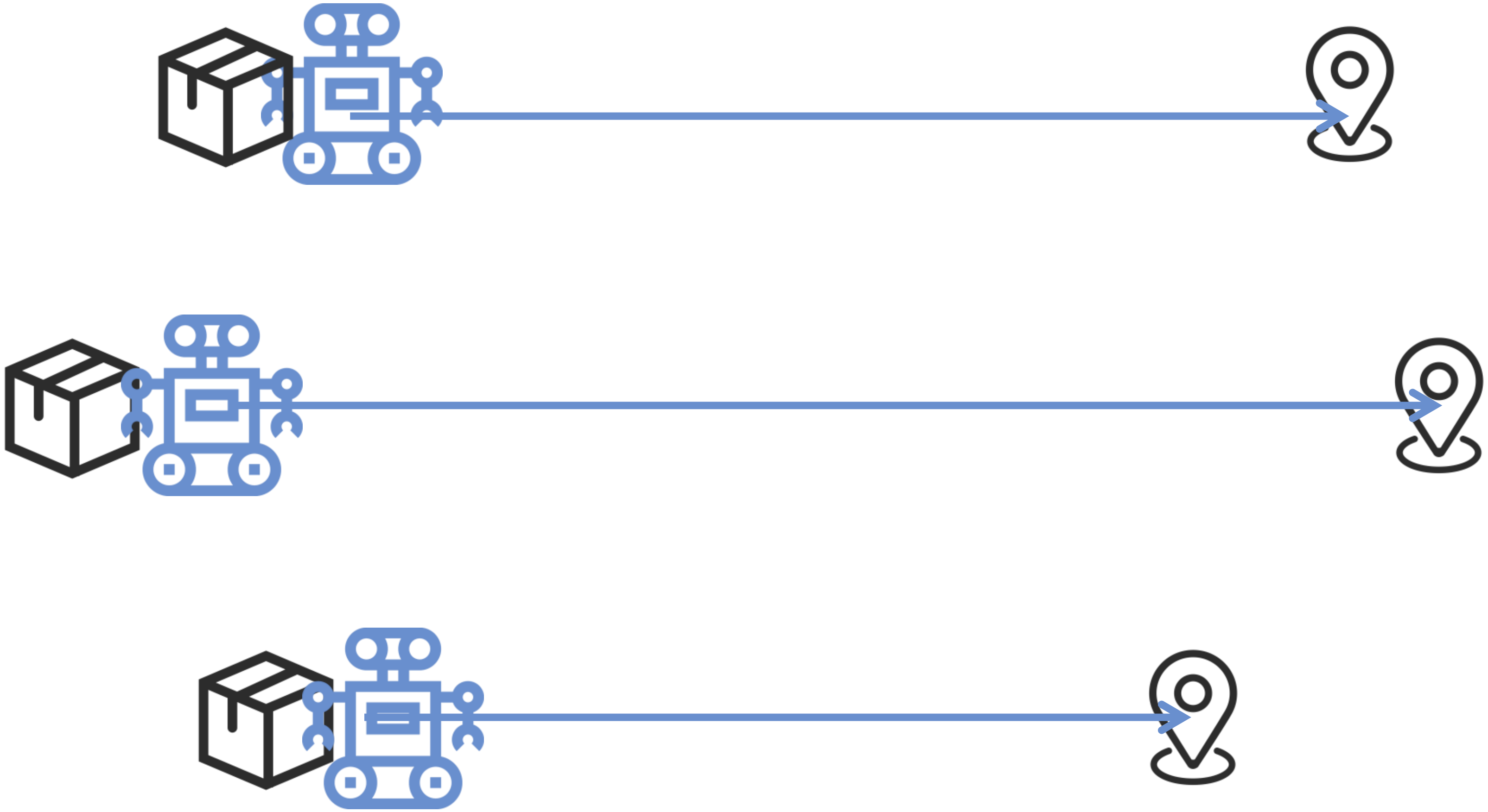

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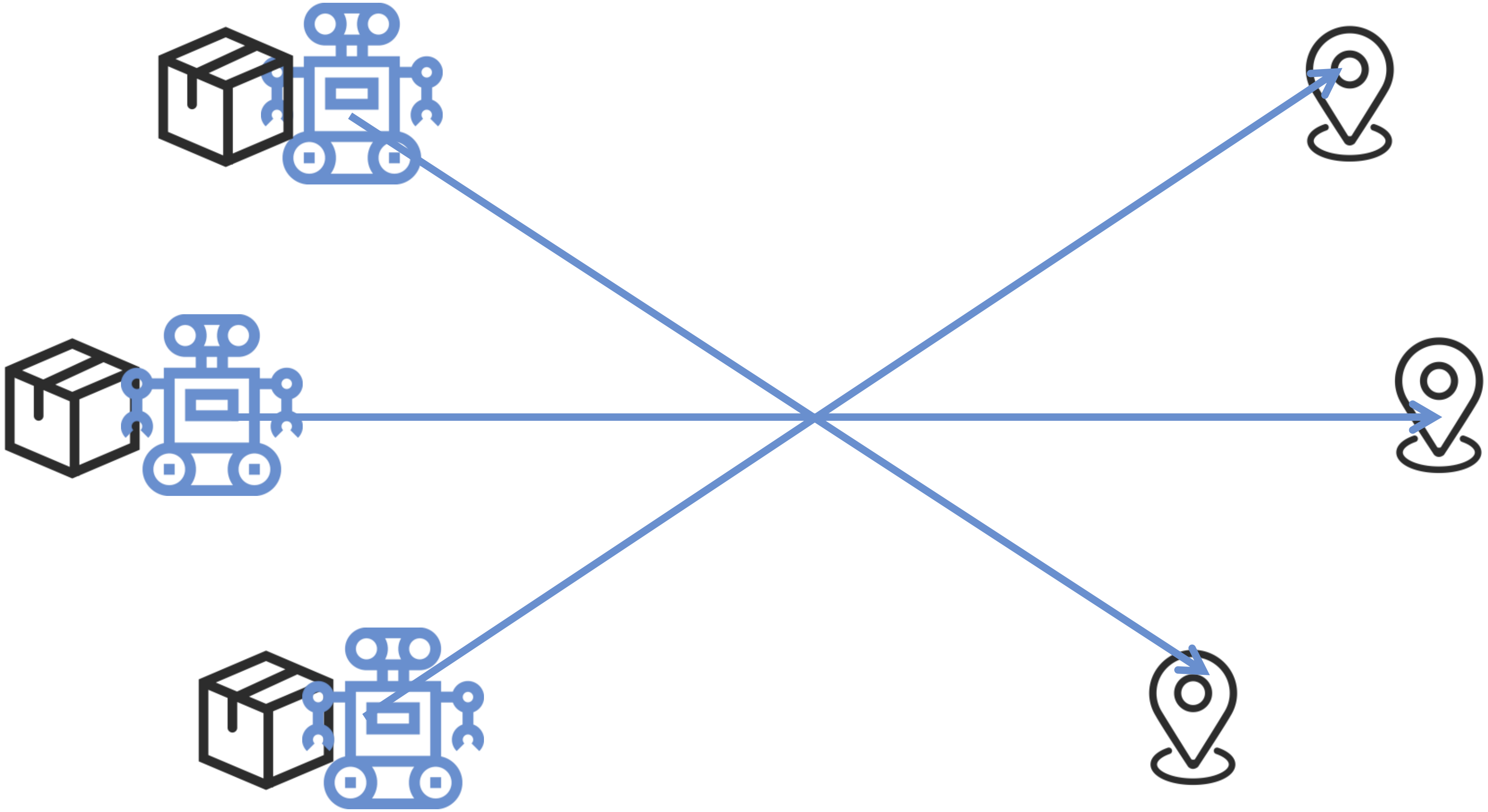

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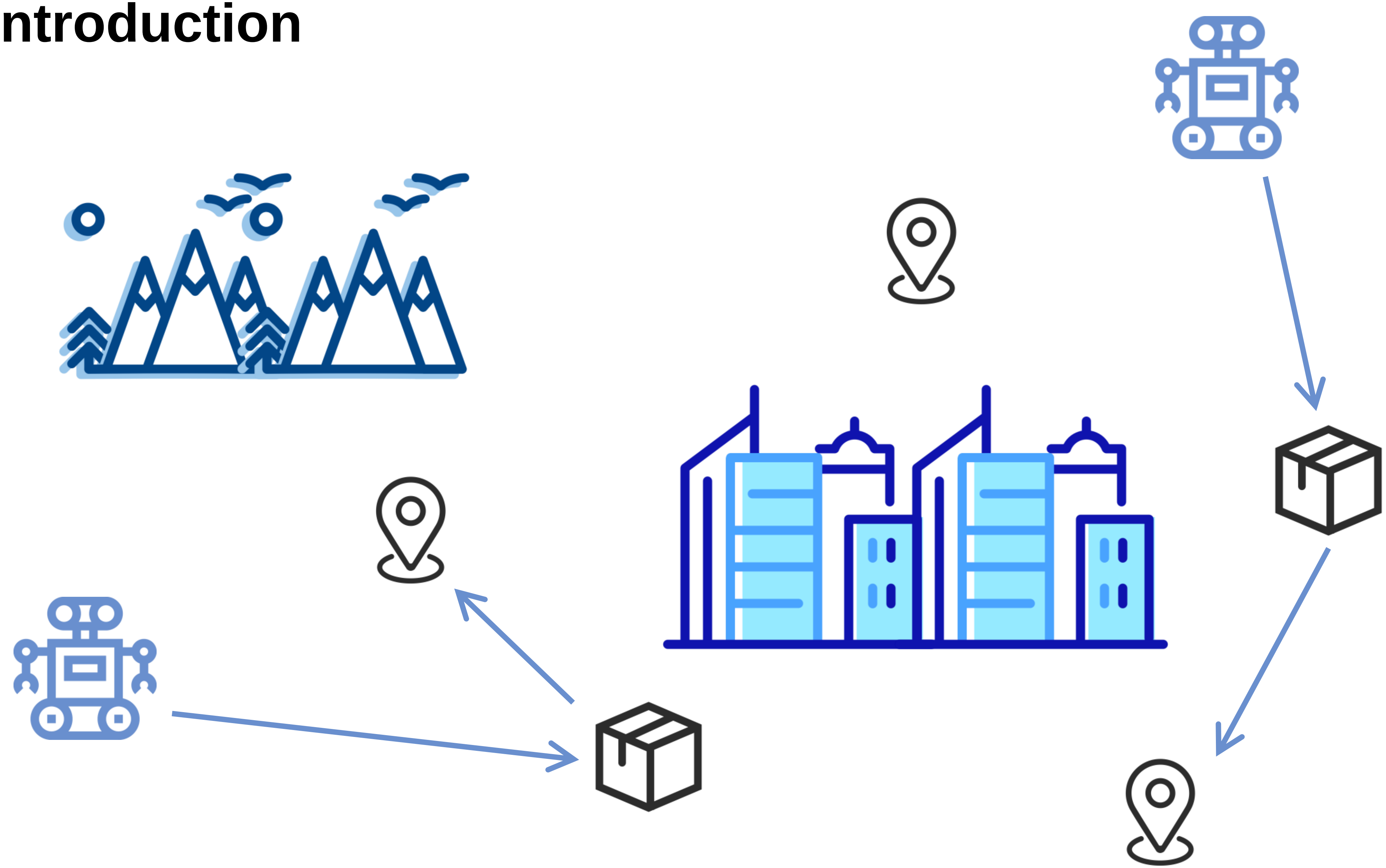

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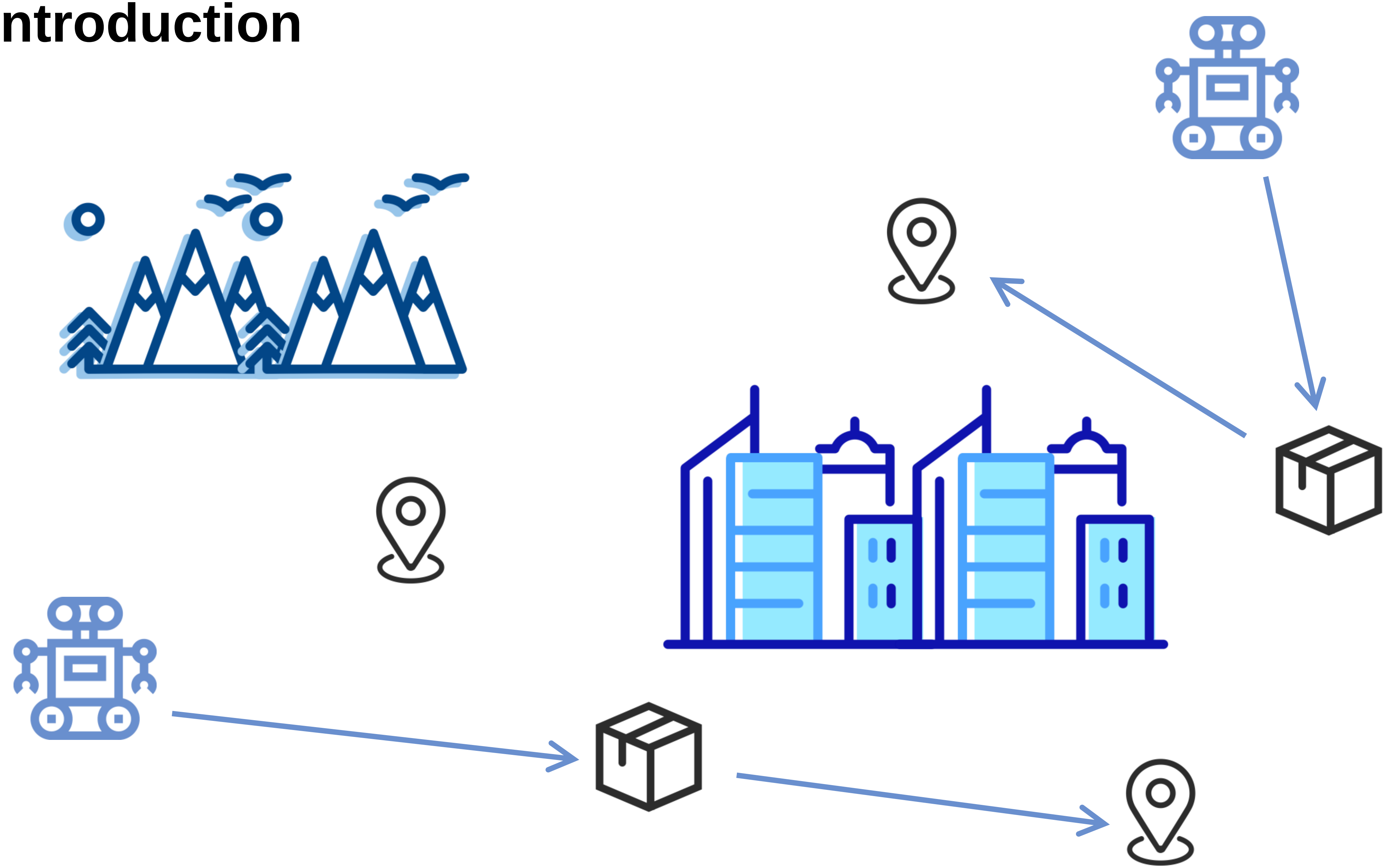
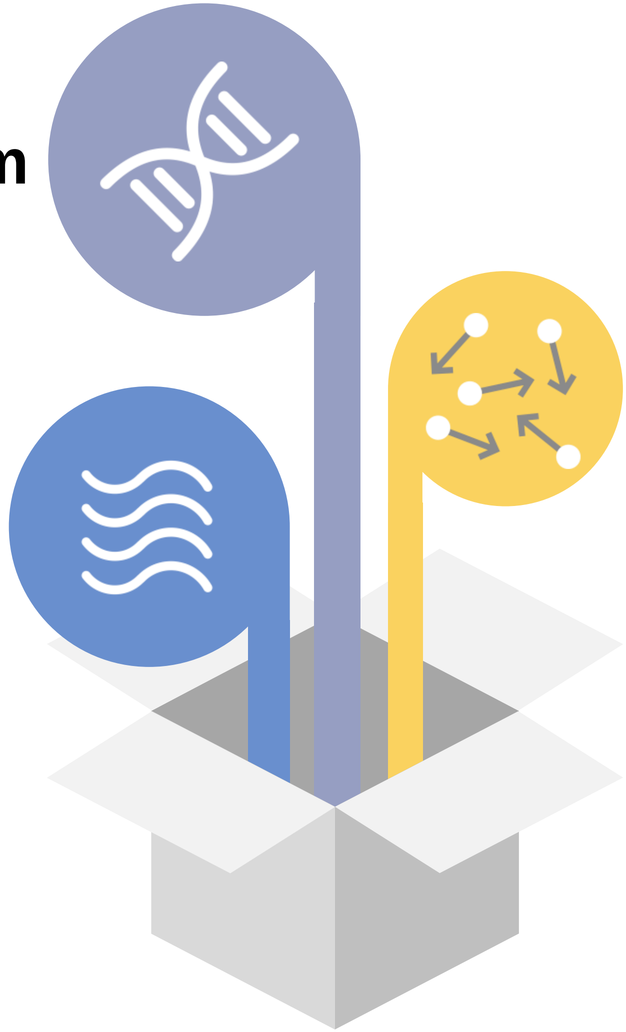


Introduction

MAPF(multi-agent path-finding)

Introduction



genetic algorithm

Particle swarm optimization

artificial potential fields

[18] Shibata, T. and Fukuda, T. (1993). Coordinative behavior in evolutionary multi-agent system by genetic algorithm.

[17] Purcaru, C., Precup, R., Iercan, D., Fedorovici, L., Petriu, E.M., and Voisan, E. (2013). Multi robot gsa- and pso-based optimal path planning instatic environments.

[19] Warren, C.W. (1990). Multiple robot path coordination using artificial potential fields.

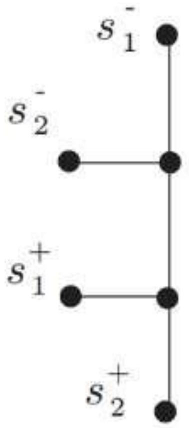
Introduction

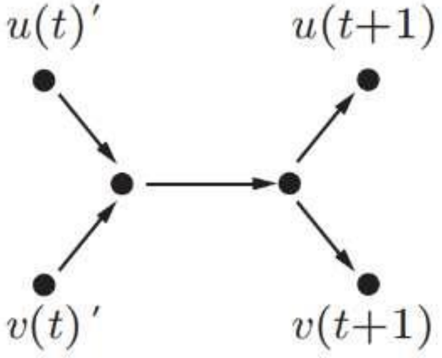
annoymous MAPF

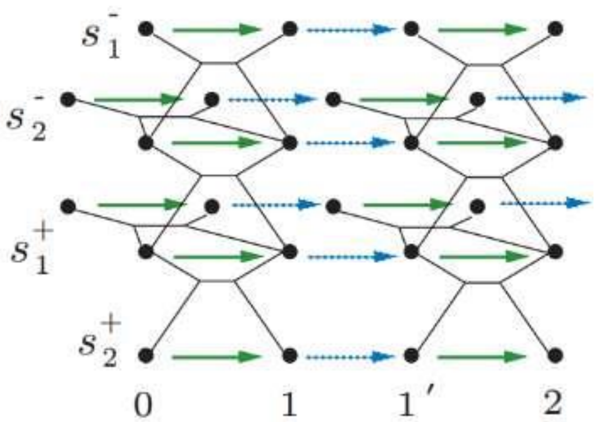
TAPF (combined target-assignment and path-finding)

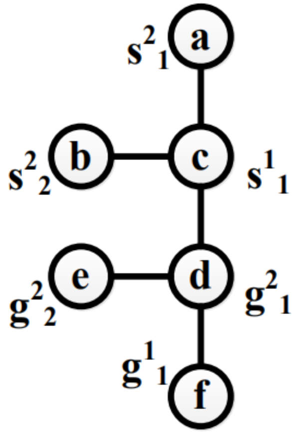
[12] Ma, H., Koenig, S.: Optimal target assignment and path finding for teams of agents(2016)

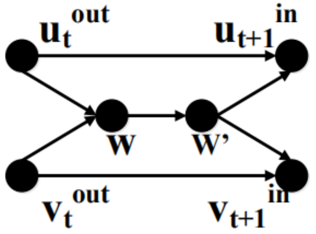
Introduction

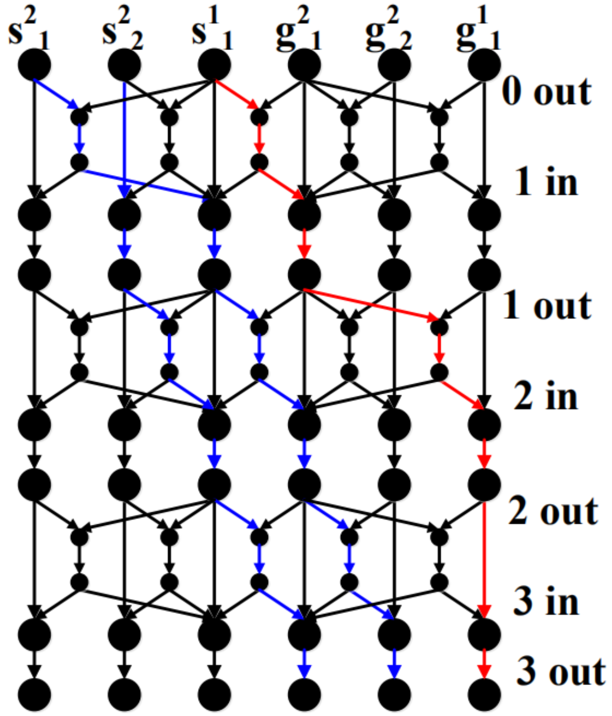












[12] Ma, H., Koenig, S.: Optimal target assignment and path finding for teams of agents(2016)

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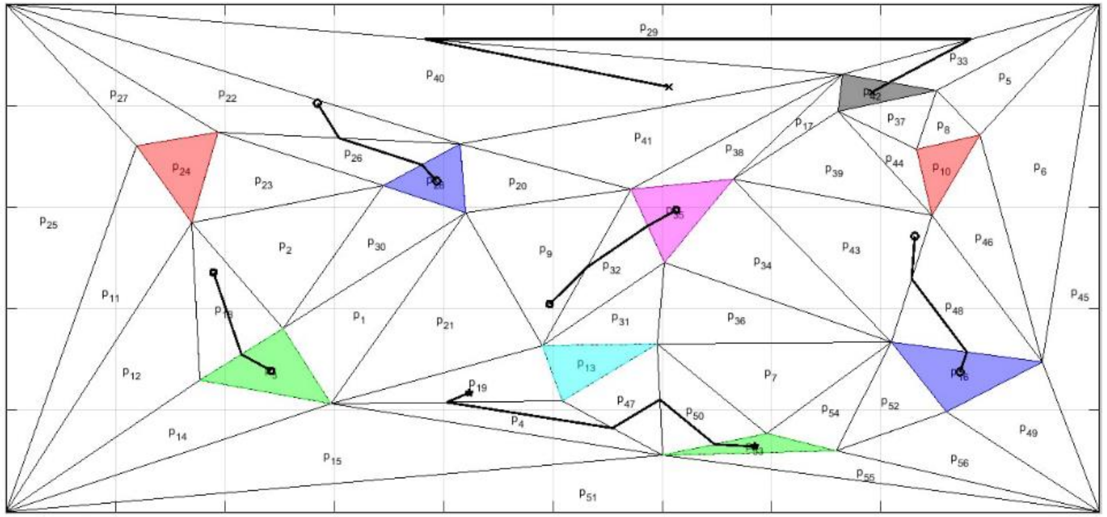


- Petri net
- Boolean specifications
- ILP (integer linear programming)

[14] Mahulea, C., Kloetzer, M.: Robot planning based on boolean specifications using Petri net models. IEEE Transactions on Automatic Control 63(7), 2218–2225 (July 2018)

Introduction

$$\min \lambda \cdot \boldsymbol{w}^T \cdot \sum_{i=1}^{k} \boldsymbol{\sigma}_i + \mu \cdot b$$

$$\begin{aligned}
\text{s.t.} \quad & \boldsymbol{m}_i = \boldsymbol{m}_{i-1} + \boldsymbol{C} \cdot \boldsymbol{\sigma}_i, i = 1, \ldots, k \\
& \boldsymbol{m}_{i-1} - \boldsymbol{Pre} \cdot \boldsymbol{\sigma}_i \geq 0, i = 1, \ldots, k \\
& \sum_{\gamma \in \mathcal{P}} (\alpha_i(\gamma) \cdot x_\gamma) \geq 1 + \sum_{\gamma \in \mathcal{P}} \min(\alpha_i(\gamma), 0), \quad \forall \varphi_i \\
& N \cdot x_\gamma \geq \boldsymbol{v}_\gamma \cdot \boldsymbol{m}_k, \quad \forall \gamma \in \mathcal{P}_f \\
& x_\gamma \leq \boldsymbol{v}_\gamma \cdot \boldsymbol{m}_k, \quad \forall \gamma \in \mathcal{P}_f \\
& N \cdot (k+1) \cdot x_\gamma \geq \boldsymbol{v}_\gamma \cdot \left(\sum_{i=0}^{k} \boldsymbol{m}_i \right), \quad \forall \gamma \in \mathcal{P}_t \\
& x_\gamma \leq \boldsymbol{v}_\gamma \cdot \left(\sum_{i=0}^{k} \boldsymbol{m}_i \right), \quad \forall \gamma \in \mathcal{P}_t \\
& \left(\boldsymbol{Post} \cdot \sum_{i=1}^{k} \boldsymbol{\sigma}_i \right) \leq b \cdot \boldsymbol{1}^T \\
& \boldsymbol{m}_i \in \mathbb{N}_{\geq 0}^{|P|}, \boldsymbol{\sigma}_i \in \mathbb{N}_{\geq 0}^{|T|}, i = 1, \ldots, k \\
& \boldsymbol{x} \in \{0,1\}^{|\mathcal{P}|}, b \geq 0.
\end{aligned}$$

→

$$\min: \ w^T \cdot \sum_{i=1}^{k} \sigma_i + \sum_{i=1}^{k} i \cdot \sum_{t \in \mathcal{T}} (\sigma_i(t))$$

$$\forall 1 \leq i \leq k, \sigma_i \in \{0,1\}^{|\mathcal{T}|},$$

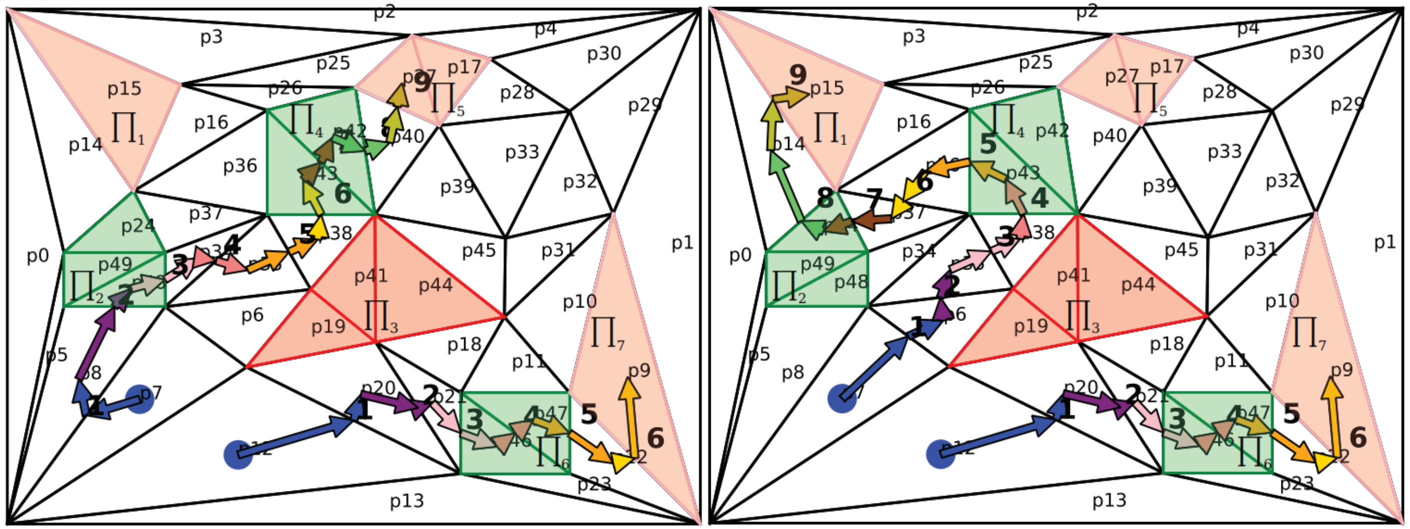
$$\forall 1 \leq i \leq k, \forall p \in \mathcal{P}_e \cup \mathcal{P}_c, m_i(p) \in \{0,1\},$$

$$\forall 1 \leq i \leq k, \forall p \in \mathcal{P}_o \cup \mathcal{P}_r, m_i(p) \in \mathbb{N},$$

$$\forall 1 \leq i \leq k, m_i = m_{i-1} + C \cdot \sigma_i,$$

$$\forall 1 \leq i \leq k, m_{i-1} - C^- \cdot \sigma_i \geq 0,$$

$$\forall p \in \mathcal{P}_o \cup \mathcal{P}_r, m_k(p) \geq 1.$$



02

Preliminaries

Preliminaries

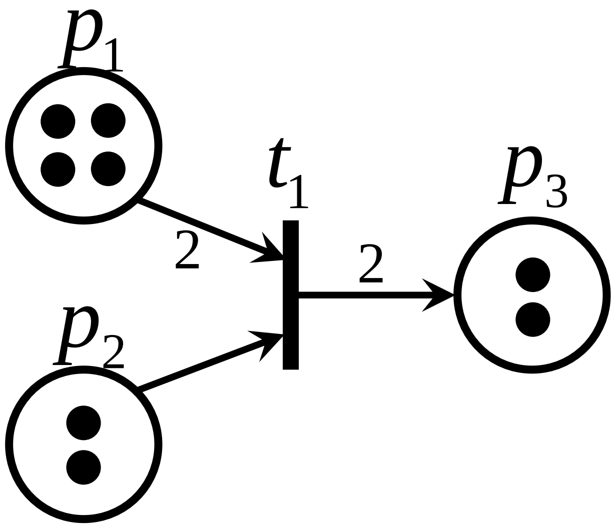
Place

Transition

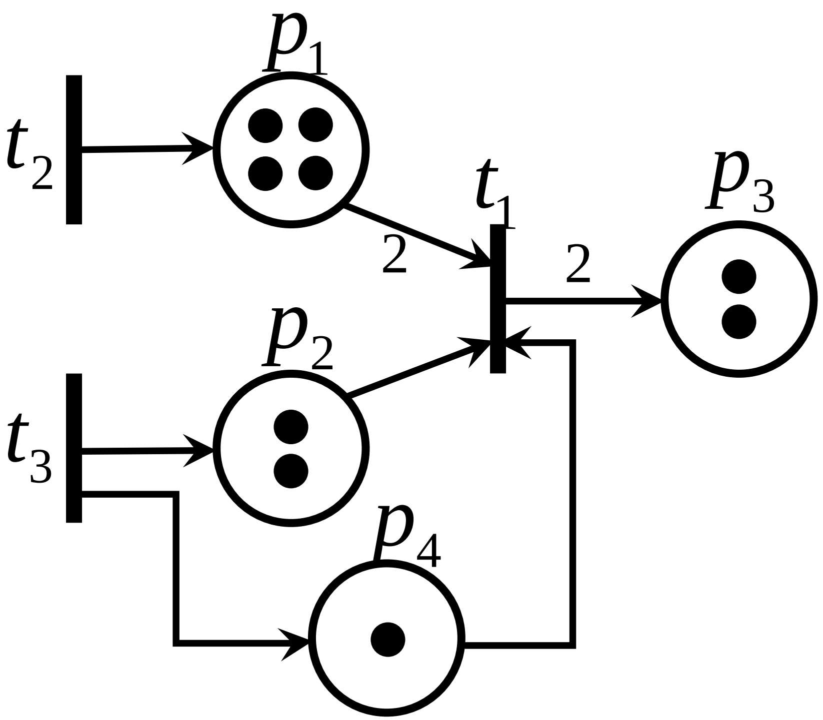
token

Arc

Preliminaries

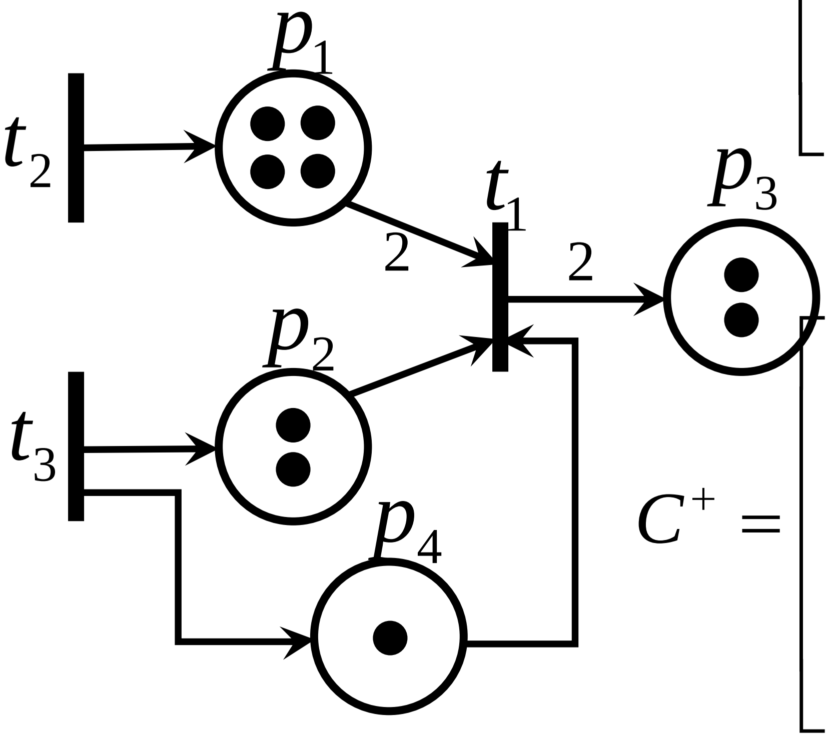


Preliminaries



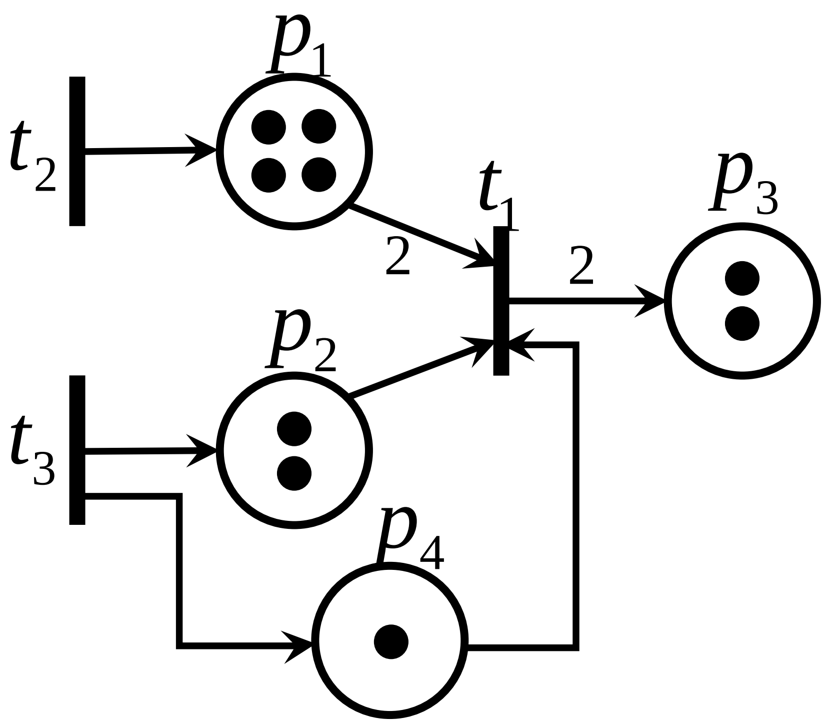
Preliminaries

$$C^- = \begin{array}{c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \left[\begin{array}{ccc} & & \\ & & \\ & & \\ & & \end{array}\right] \end{array} \begin{array}{c} p_1 \\ p_2 \\ p_3 \\ p_4 \end{array}$$



$$C^+ = \left[\begin{array}{ccc} & & \\ & & \\ & & \\ & & \end{array}\right] \begin{array}{c} p_1 \\ p_2 \\ p_3 \\ p_4 \end{array}$$

Preliminaries



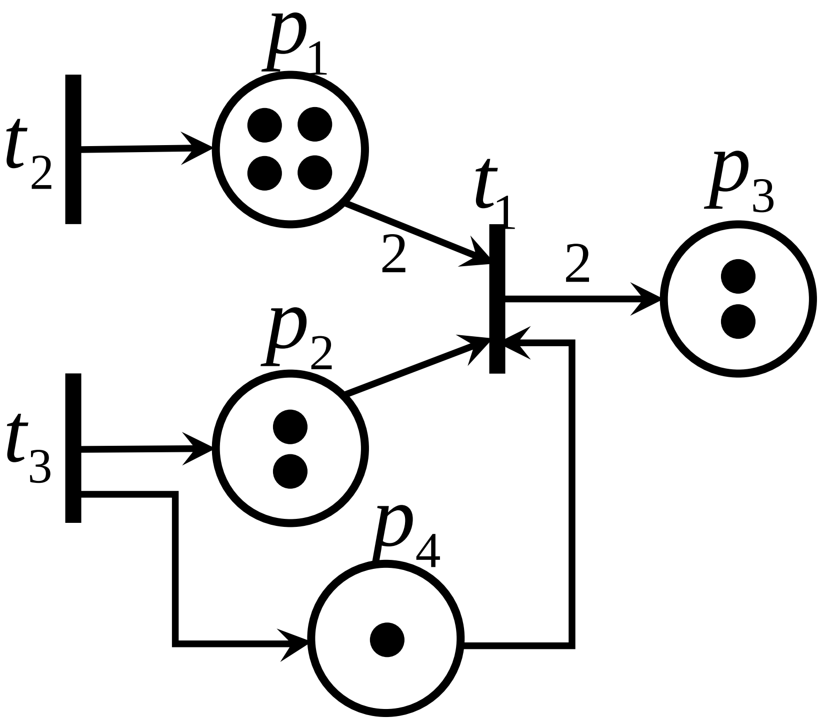
$$C^- = \begin{array}{c@{}c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} & \\ \begin{bmatrix} 2 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} & \begin{array}{c} p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \end{array}$$

$$C = C^+ - C^-$$

$$= \begin{bmatrix} -2 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{array}{c} p_1 \\ p_2 \\ p_3 \\ p_4 \end{array}$$

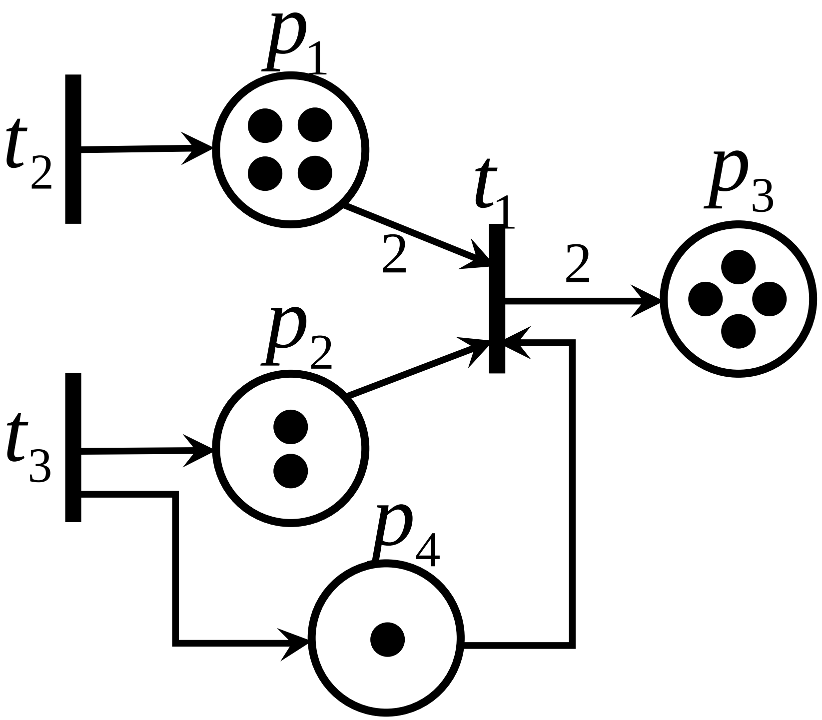
$$C^+ = \begin{array}{c@{}c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} & \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{array}{c} p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \end{array}$$

Preliminaries



$$m = \begin{bmatrix}4\\2\\2\\1\end{bmatrix} \begin{array}{c} p_1\\p_2\\p_3\\p_4\end{array} + \begin{bmatrix} -2 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix}1\\0\\0\end{bmatrix}$$

Preliminaries



$$m = \begin{bmatrix} 2 \\ 1 \\ 4 \\ 0 \end{bmatrix}$$

Preliminaries

$$\forall 1 \leq i \leq k, \sigma_i \in \{0,1\}^{|\mathcal{T}|},$$

$$\forall 1 \leq i \leq k, m_i = m_{i-1} + C \cdot \sigma_i,$$

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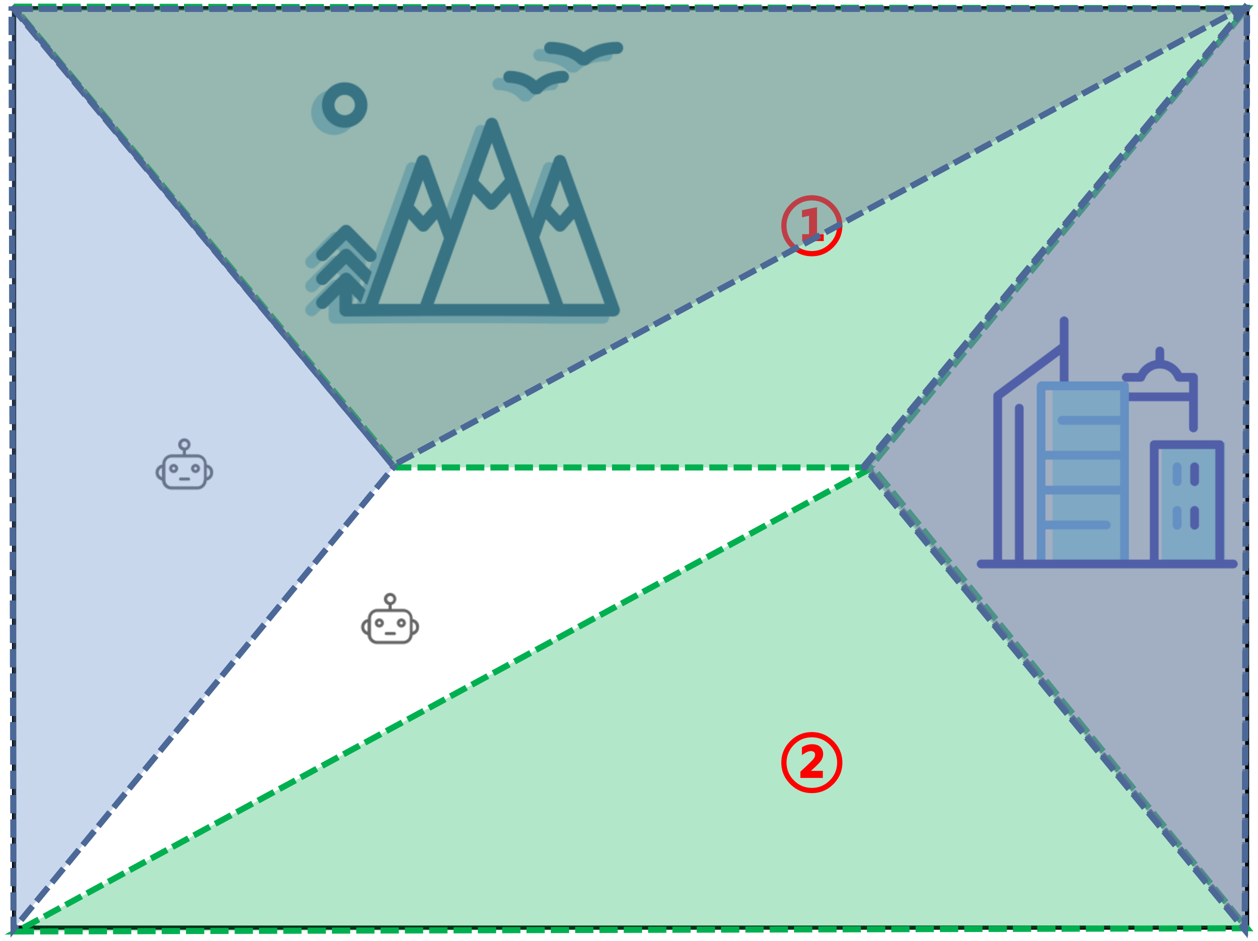
03

Problem Descriptions and PN Modeling Method

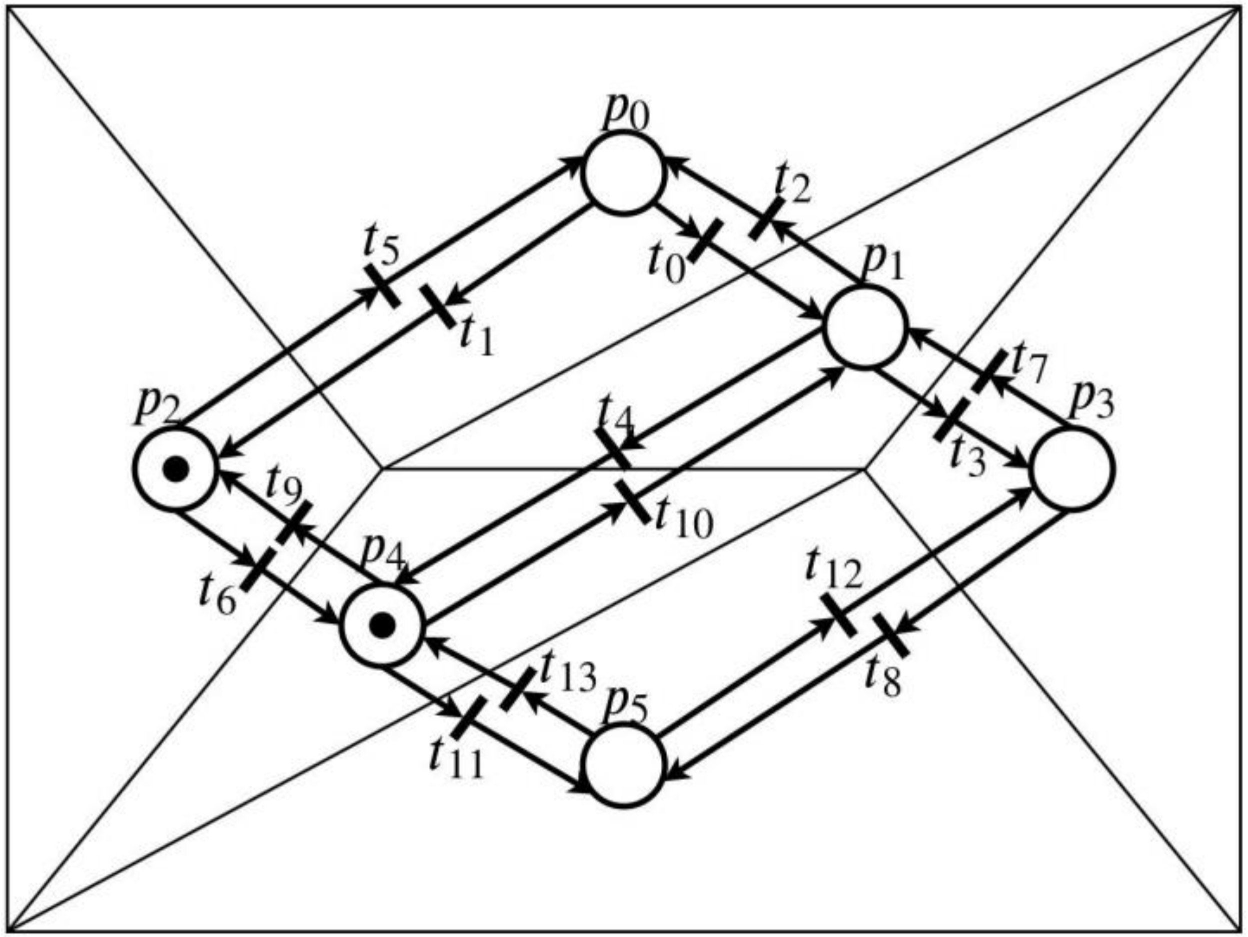
Problem Descriptions



🟩	must pass through	V
🟥	①then②	τ
🟦	eventually stay	G
◼	forbidden	b

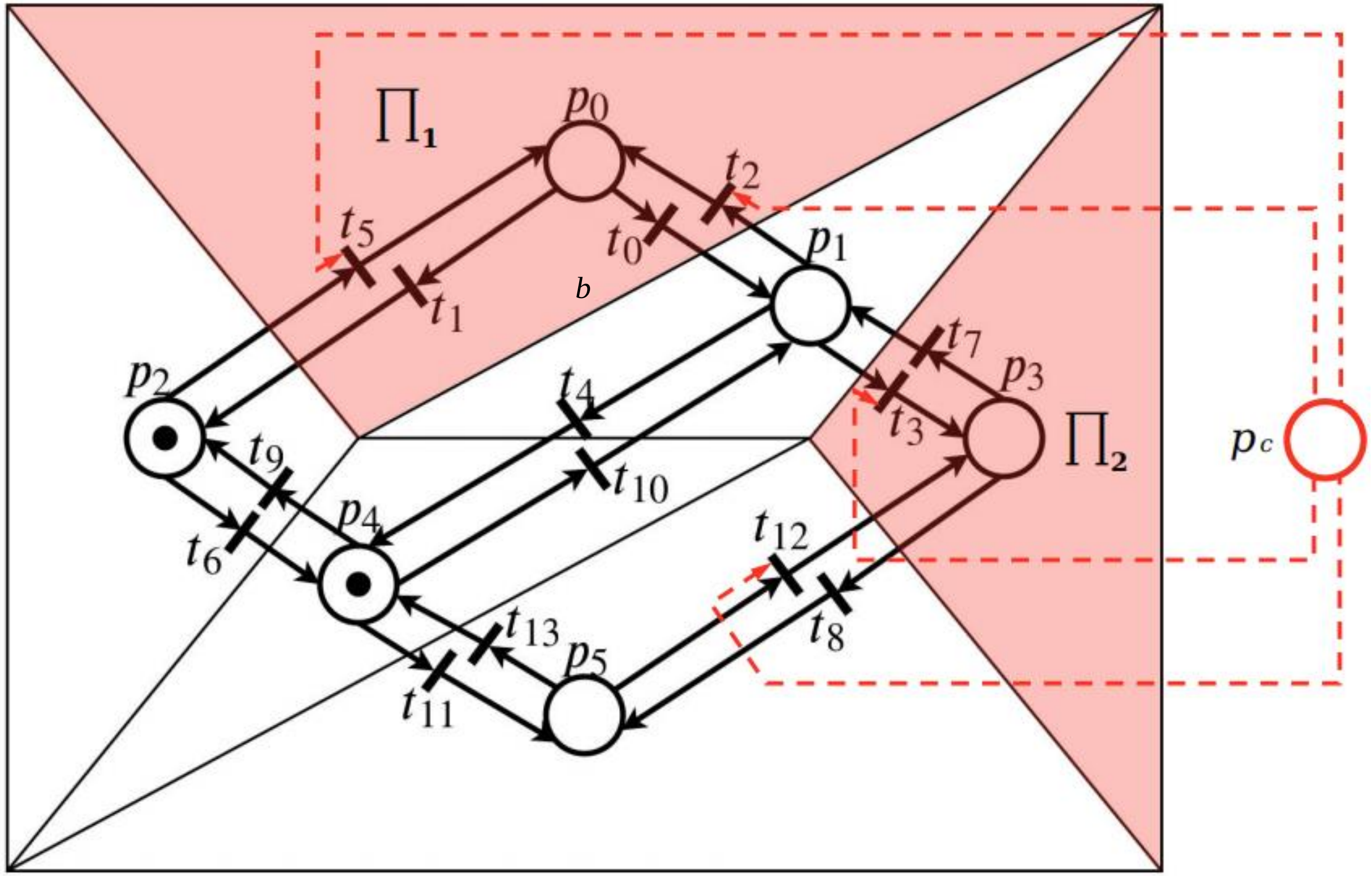
PN model for a multi-robot system

PN model for a multi-robot system



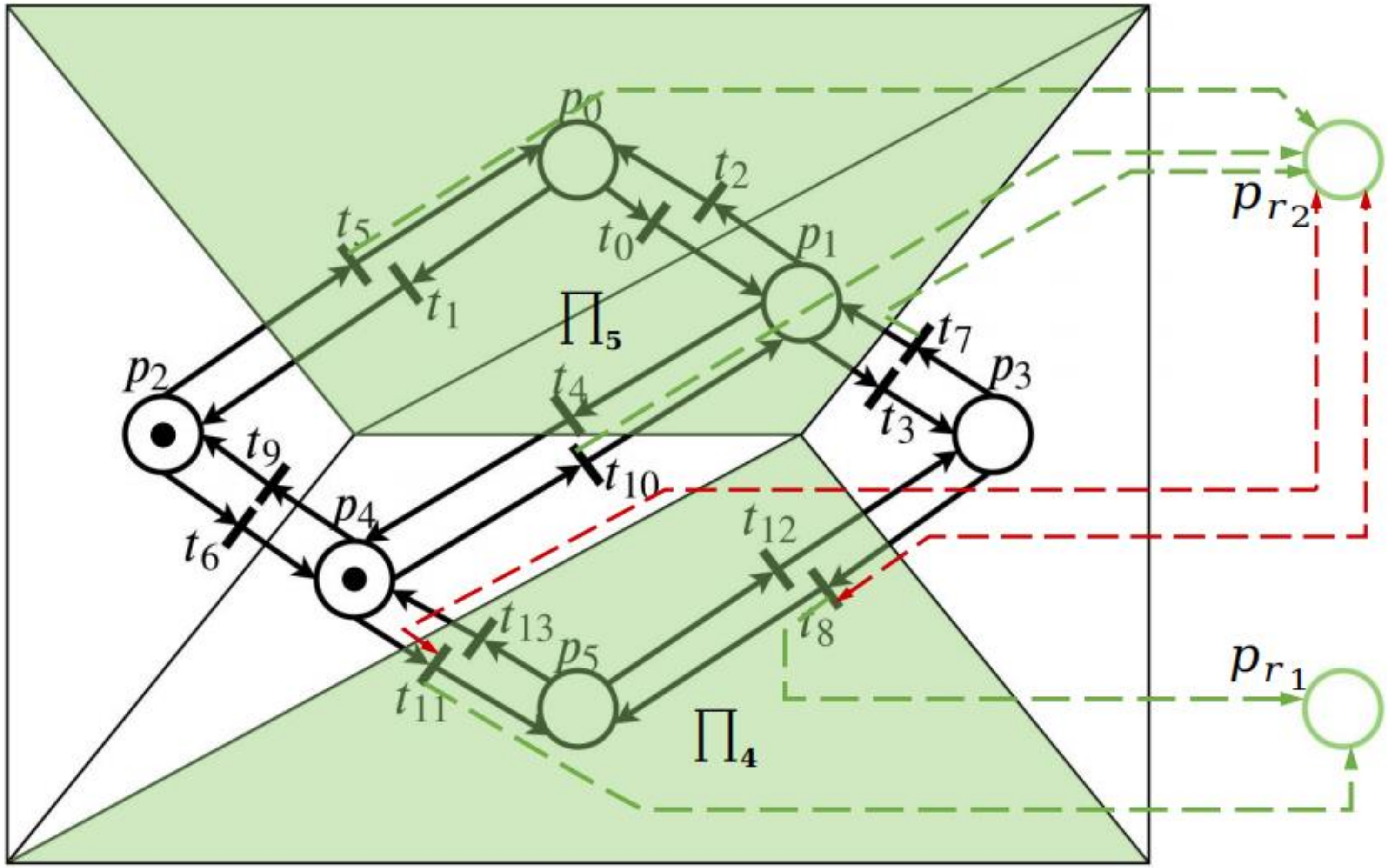
Synthesis method of control places enforcing the sub-specification b

Add restrictions on prohibited areas



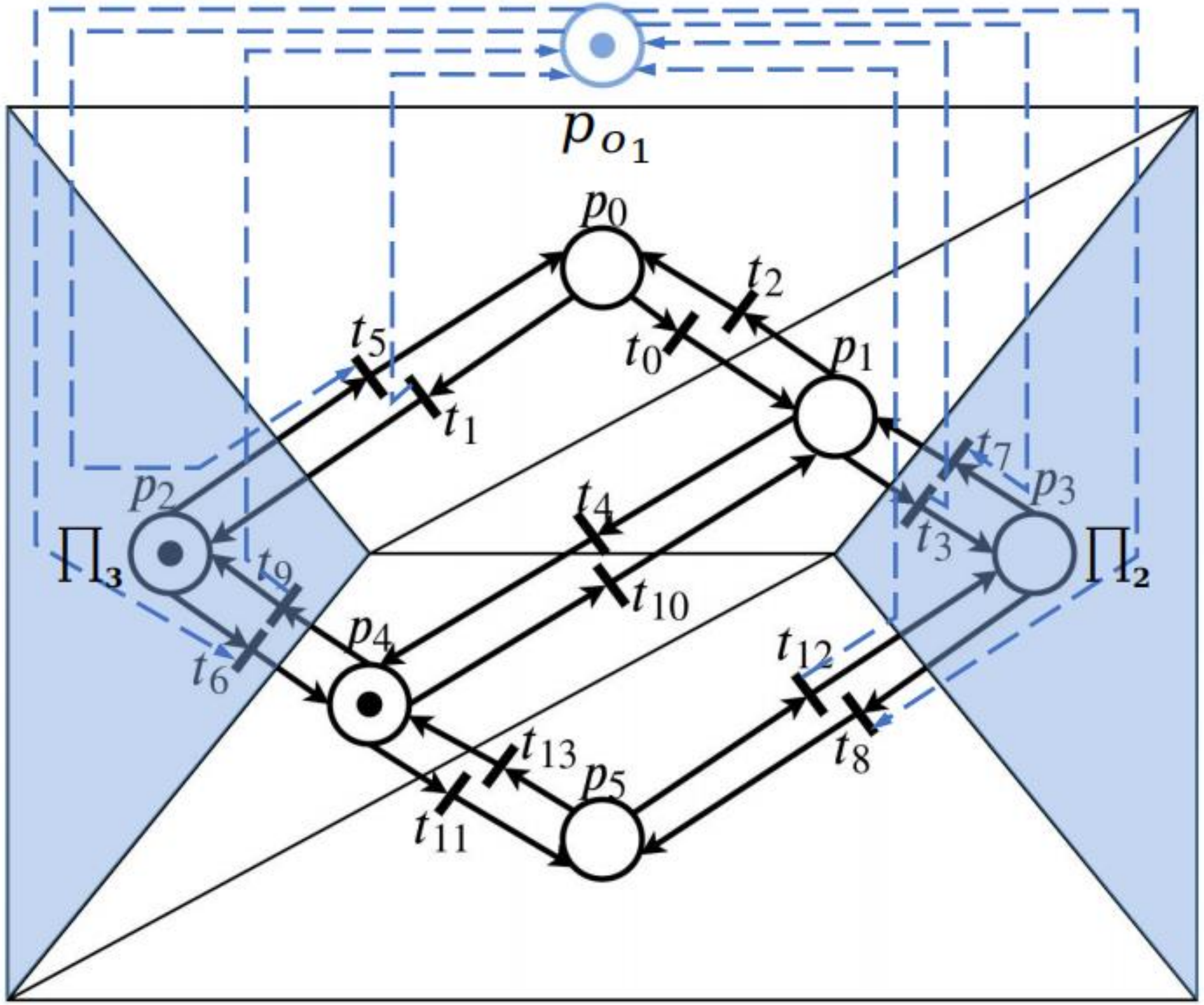
Synthesis method of recording places enforcing the sub-specification V and τ

Add recording places to the areas that must be passed, and restrict the order.



Synthesis method of observing places enforcing the sub-specification G

Add observing places to the area where the robot will eventually stay



04

ILP problem obtained by PNs

ILP problem obtained by PNs

$$\min: \ w^T \cdot \sum_{i=1}^{k} \sigma_i + \sum_{i=1}^{k} i \cdot \sum_{t \in \mathcal{T}} \left(\sigma_i(t)\right)$$

$$\forall 1 \leq i \leq k, \sigma_i \in \{0,1\}^{|\mathcal{T}|},$$

$$\forall 1 \leq i \leq k, \forall p \in \mathcal{P}_e \cup \mathcal{P}_c, m_i(p) \in \{0,1\},$$

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$$\forall 1 \leq i \leq k, m_i = m_{i-1} + C \cdot \sigma_i,$$

$$\forall 1 \leq i \leq k, m_{i-1} - C^- \cdot \sigma_i \geq 0,$$

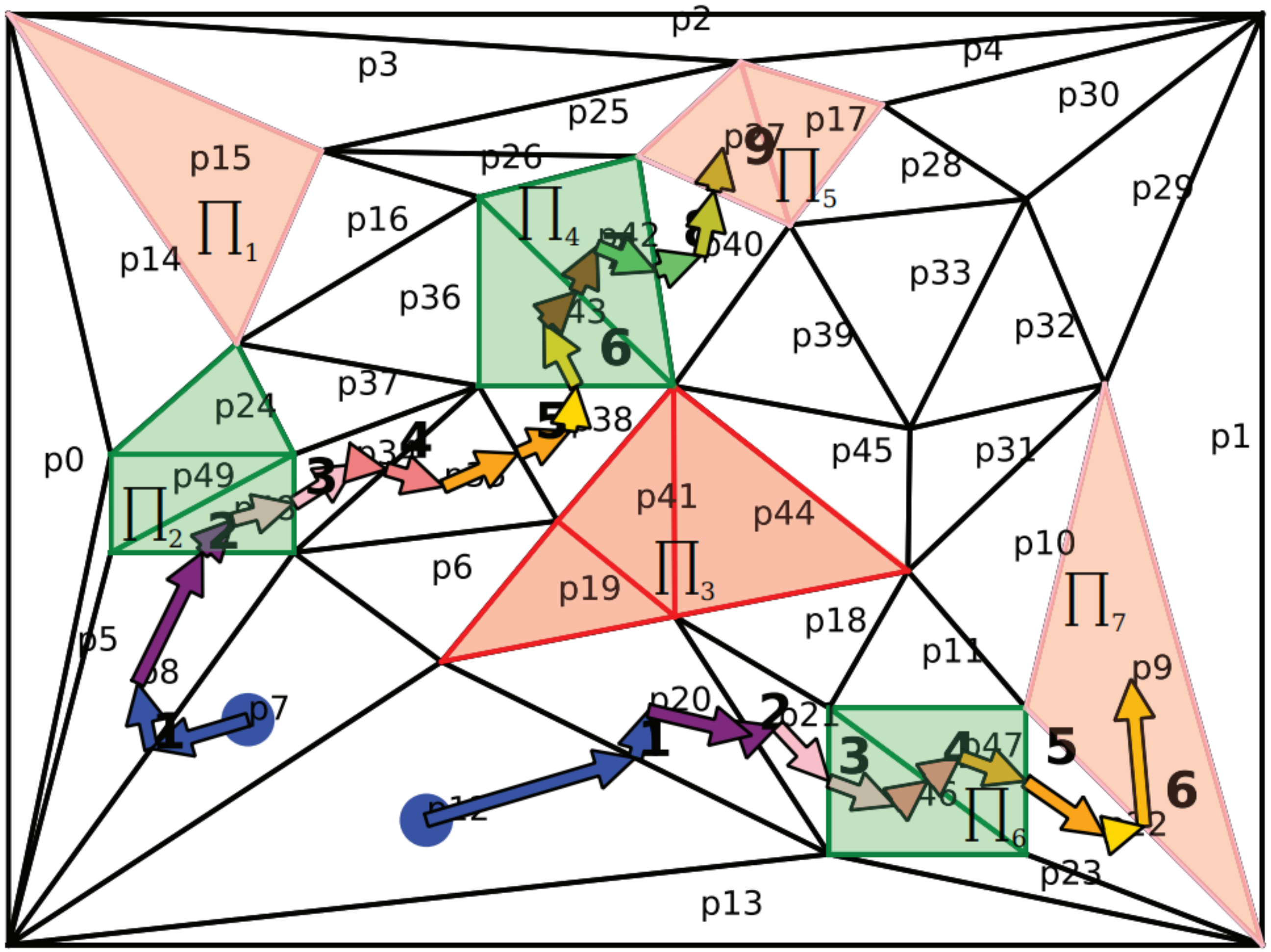
$$\forall p \in \mathcal{P}_o \cup \mathcal{P}_r, m_k(p) \geq 1.$$

05

Numerical Experiments

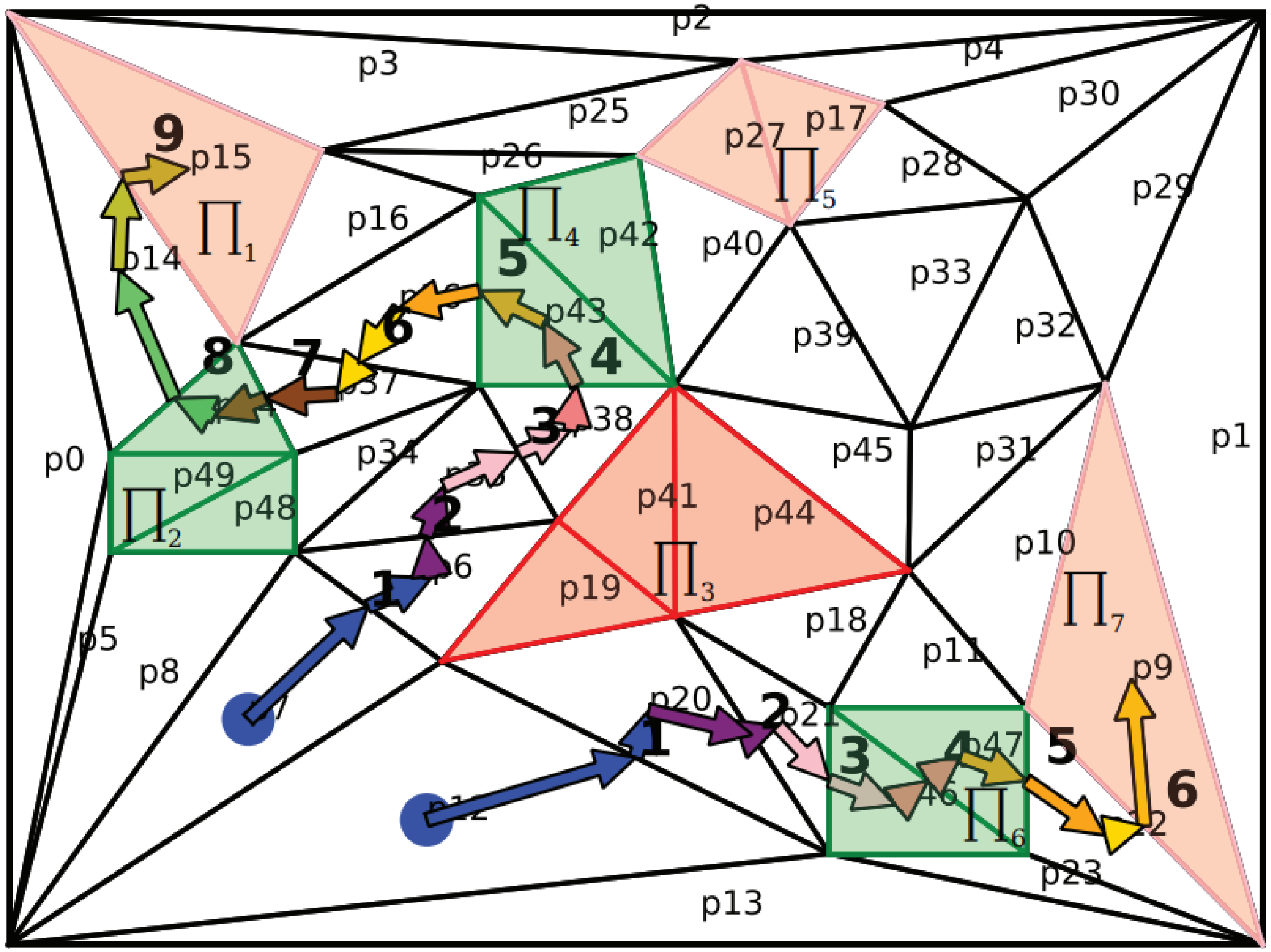
Numerical Experiments

$$\varphi_1 = \neg\Pi_3 \wedge \Pi_2 \wedge \Pi_4 \wedge \Pi_6 \wedge (\pi_1 \vee \pi_5) \wedge \pi_7,$$

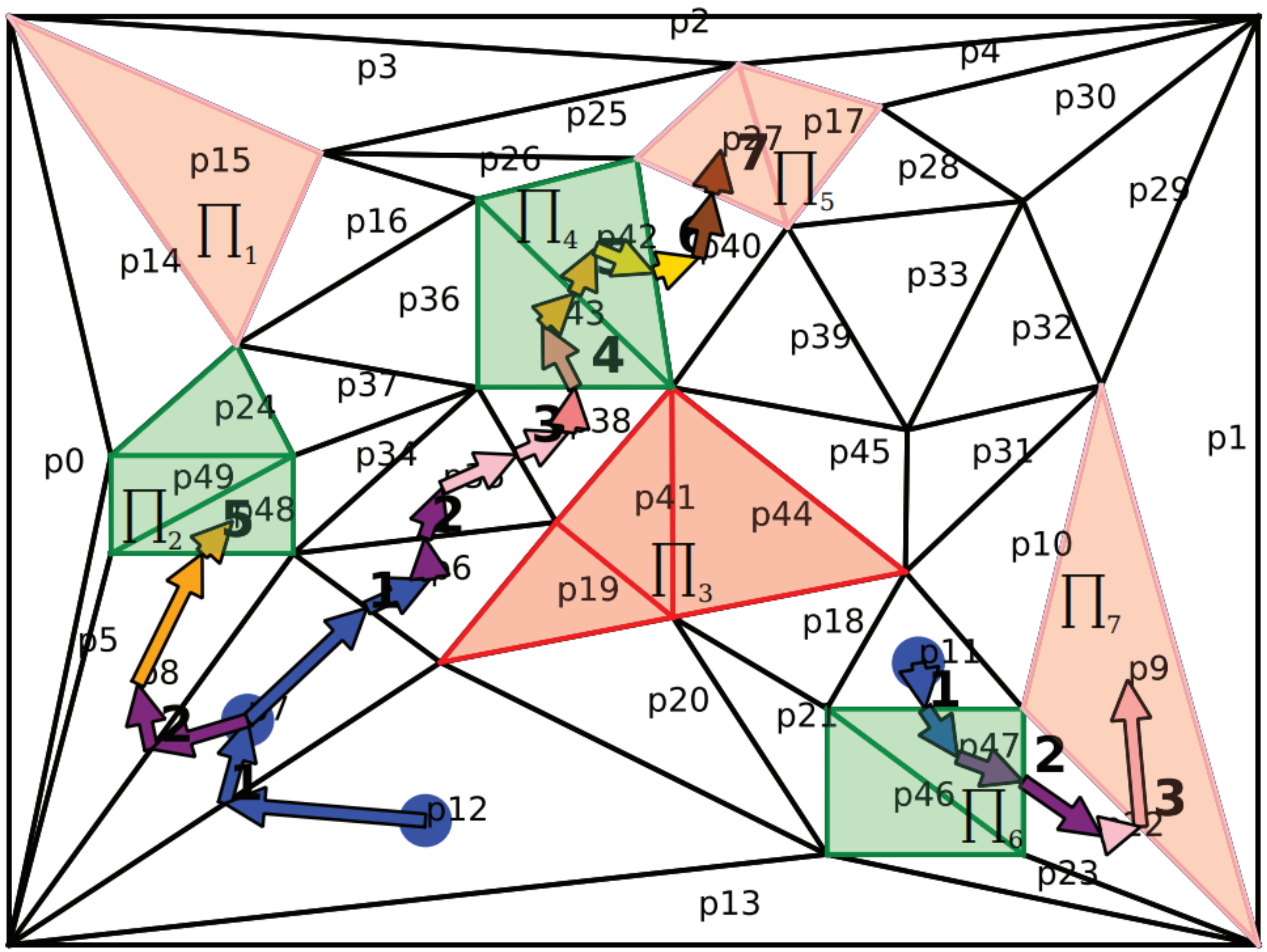


Numerical Experiments

$$\varphi_2 = \varphi_1 \wedge (\neg \Pi_2 \ \mathbf{u} \ \Pi_4)$$



Numerical Experiments



Thanks.